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Wave interaction with multiple articulated floating elastic plates

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Abstract

Flexural gravity wave scattering by multiple articulated floating elastic plates is investigated in the three cases for water of finite depth, infinite depth and shallow water approximation under the assumptions of two-dimensional linearized theory of water waves. The elastic plates are joined through connectors, which act as articulated joints. In the case when two semi-infinite plates are connected through a single articulation, using the symmetric characteristic of the plate geometry and the expansion formulae for wave-structure interaction problem, the velocity potentials are obtained in closed forms in the case of finite and infinite water depths. On the other hand, in the case of shallow water approximation, the continuity of energy and mass flux are used to obtain a system of equations for the determination of the full velocity potentials for wave scattering by multiple articulations. Further, using the results for single articulation and assuming that the articulated joints are wide apart, the wide-spacing approximation method is used to obtain the reflection coefficient for wave scattering due to multiple articulated floating elastic plates. The effects of the stiffness of the connectors, length of the elastic plates and water depth on the propagation of flexural gravity waves are investigated by analysing the reflection coefficient.

Keywords: Multiple articulation; Linearized theory of water waves; Elastic plate; Shallow water approximation; Wide-spacing approximation

1. Introduction

In the recent past, there has been significant progress in the hydroelastic analysis of very large floating structures (VLFS) which are meant for ocean space utilization for various human activities. The unique characteristics of these types of ocean structures are primarily related to their unprecedented length, displacement and associated hydroelastic response, analysis and design. These large structures consist of many modules, which are fabricated in shipyards and then articulated together on site. The articulation of the elastic plates is done by the connectors which depend on the stiffness constants known as the vertical linear spring stiffness and flexural rotational spring stiffness. Xia et al. (2000) analysed the hydroelastic behaviour of an articulated plate by modelling the connectors by singular line loading and its derivatives in water of finite depth. To reduce the vibration of the floating structure, using two different physical

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approaches Khabakhpasheva and Korobkin (2002) analysed the hydroelastic behaviour of the compound floating plates under the influence of surface waves. In the first approach, an auxiliary spring-and-mass system was added to reduce the vibration of the main structure. In this approach an elastic plate of smaller size compared to the original floating elastic plate was connected to the original structure with torsional spring stiffness. However, in the second approach, the floating structure was connected to the sea bottom with a spring. Chung and Fox (2005) described various transition conditions at the joint of two semi-infinite floating elastic plates and obtained a solution by the Wiener-Hopf technique to investigate the scattering of oblique flexural gravity wave propagation across a crack in a floating ice-sheet in water of finite depth. Karmakar and Sahoo (2005) studied the wave scattering by a single articulated floating elastic plate in water of infinite depth by a direct application of a mixed type Fourier transform and orthogonal mode-coupling relation where all unknowns were obtained in terms of convergent integrals. Karmakar et al. (2007) re-derived the expansion formulae of Manam et al. (2006) and analysed the effect of single articulation and compression on the scattering of flexural gravity waves in infinite water depth. Fu et al. (2007) examined the hydroelastic response of flexible floating interconnected structures based on the finite element method. In addition, significant progress in the literature on hydroelasticity theories of VLFS are reviewed by Kashiwagi (2000), Watanabe et al. (2004) and Chen et al. (2006). Suzuki et al. (2006) provided a detailed overview of the history, application and uniqueness of VLFS including the design, construction, and the future scope of work.

A very significant amount of progress on wave-ice interaction has been made in the literature using the floating elastic plate model, which finds application in the field of cold region science and technology, for the large sheets of ice that cover a vast area of the ocean surface in the Arctic and Antarctic regions. Squire and Dixon (2001) developed a model based on the application of Green's function to allow interaction of normally incident ice-coupled waves with any number of cracks. Williams and Squire (2004) studied the oblique wave scattering of plane flexural gravity waves due to randomly shaped and spaced irregularities in sea-ice by the application of Green's function and wide-spacing approximation. Porter and Evans (2006) studied the scattering of flexural gravity waves by multiple narrow cracks in ice-sheets by using canonical source function approach and wide-spacing approximation in which the ice-sheet was modelled as a thin elastic plate. The wave reflection by a semi-infinite periodic array of cracks was formulated exactly in terms of a convergent infinite system of equations. Williams and Squire (2006) developed a theoretical model to describe the wave propagation through three floating elastic plates based on the methods of Wiener-Hopf technique and the residue calculus technique with the edges being either fixed or free. Manam et al. (2006) derived the general expansion formulae and related mode-coupling relations based on the Fourier analysis to tackle a general class of boundary value problems and applied them to study the scattering of ice-coupled waves by a straight crack in infinite water depth. Porter and Evans (2007) considered the diffraction of flexural gravity waves by multiple cracks of finite lengths in an elastic sheet and used the Fourier transform to obtain a system of hypersingular integral equations, which were solved by Galerkin's method. Kohout et al. (2007) studied the linear wave propagation through multiple floating elastic plates of variable properties based on the method of eigenfunction expansion and appropriate matching conditions. Vaughan et al. (2007) analysed the scattering of ice-coupled waves by imperfections in an ice-sheet, examining the cracks and pressure ridges, and obtained asymptotic solutions to physically analyse the zeroes present in the reflection coefficient. Kohout (2008) in her Ph.D. thesis investigated the water wave scattering by floating elastic plates with applications to sea-ice. Squire (2008) discussed the synergy between very large floating structures and sea-ice interaction with surface gravity waves. Bennetts et al. (2009) analysed the scattering of flexural gravity waves with periodic structures and observed that the variations in the periodic irregularities produce strong resonances about the 'Bragg frequencies' for relatively few periods.

Most of these investigations on wave interaction with floating elastic plates/ice-sheets are based on small amplitude wave theory in water of finite or infinite depths. On the other hand, a majority of the VLFS are constructed near the shoreline, where the water depths are relatively shallow. Marchenko and Voliak (1997) analysed the scattering of flexural gravity waves in shallow fluid beneath an ice cover with linear irregularities due to cracks and hummocks. Sturova (2001) analysed the deflection of floating flexible platforms in shallow water after reducing the problem to a system of boundary integral equations. Andrianov and Hermans (2003) discussed the influence of water depth on the hydroelastic response of very large floating structures. Ohkusu and Namba (2004) studied the hydroelastic behaviour of large floating structures using shallow water theory. Sturova (2006) analysed the steady-state behaviour of a floating elastic plate acted upon by a localized external load using linear shallow water theory.

In the present study, scattering of flexural gravity waves by multiple articulated floating elastic plates is investigated and compared in the three cases of finite water depth, infinite water depth, and shallow water approximation. Multiple plates are assumed to be connected by articulation through vertical linear springs and flexural rotational springs. The two end plates are assumed to be infinitely extended, thus covering the entire water surface. Applying the generalized expansion formulae for wave-structure interaction problems as developed in Manam et al. (2006) and utilizing the continuity and edge conditions, explicit solutions in the case of a single articulation are obtained for both finite and infinite water depths. The results of a single articulation are then used to obtain the results for multiple articulations using the wide-spacing approximation method. The derivations of the velocity potentials for multiple articulations in both the cases of finite and infinite water depths are discussed in brief to give an idea about the complexity of the problem in the direct method compared to that of the wide-spacing approximation method. However, in the case of shallow water approximation, the results for multiple articulations are also computed directly by using the continuity of energy and mass flux at the articulated edges. The numerical results and discussion are based on the analysis of reflection coefficient, which provides a qualitative insight on the reflection of the flexural gravity waves due to multiple articulations. The effects of change in water depth, plate length and the values of the stiffness constants are studied in the case of plates having single, double and four articulations.

2. Wave scattering by articulated floating elastic plate

In this section, the general mathematical formulation and the solution procedures in specific cases associated with wave scattering by multiple articulated floating elastic plates in water of finite depth, infinite depth and shallow water approximation are described in detail.

2.1. The general boundary value problem

In the present study, a two-dimensional Cartesian coordinate system is chosen, with the x-axis being horizontal and the y-axis being vertical positive downward, as shown in Fig. 1. The fluid is assumed to occupy the region $-\infty<x<\infty$, $0<y<h$ in the case of finite water depth and $-\infty<x<\infty$, $0<y<\infty$ in the case of infinite water depth. It is assumed that the undisturbed mean water surface $y=0$, $-\infty<x<\infty$, is covered by an elastic plate of thickness d, which is a combination of multiple floating elastic plates joined through proper articulations. Without any loss of generality, a total of $N+1$ plates with N articulations at $x=-a_j$, $y=0$, $j=1,2,\ldots,N$, are considered as shown in Fig. 1. The whole domain is divided into $N+1$ regions along the vertical interfaces at the articulated edges $x=-a_j$ and are denoted by I_j, where $I_j \equiv (-a_j<x<-a_{j-1})$ for $j=2,3,\ldots,N$ and $I_1 \equiv (-a_1<x<\infty)$, $I_{N+1} \equiv (-\infty<x<-a_N)$, with $0<y<h$ in the case of finite water depth and $0<y<\infty$ in the case of infinite water depth. It may be noted that all the plates are of finite length, except the two plates at the two far ends, which are assumed to be of semi-infinite length as in Fig. 1.

It is assumed that a monochromatic flexural gravity wave is normally incident from the positive x direction on the first articulation at $x=-a_1$ and propagates through the multiple articulated plates. It experiences partial reflection and transmission at each and every articulated edge before being transmitted into the last semi-infinite plate region. Assuming that the fluid is inviscid and incompressible, and the motion is irrotational and simple harmonic in time with angular frequency ω, the velocity potential $\Phi_j(x,y,t)$ is expressed in the form $\Phi_j(x,y,t)=\mathscr{R}e\{\phi_j(x,y)\mathrm{e}^{-\mathrm{i}\omega t}\}$, where $\mathscr{R}e$ denotes the real part and the subscript j refers to the respective regions. The spatial velocity potential $\phi_j(x,y)$ satisfies the governing equation

$$\nabla^2\phi_j=0 \quad \text{in the fluid domain.} \tag{1}$$

Considering the Euler–Bernoulli beam model for the elastic plates, the linearized plate boundary condition on the mean free surface is given by [as in Karmakar and Sahoo (2005)]

$$\{(1+D\partial_x^4)\partial_y+K\}\phi_j=0 \quad \text{on } y=0,\ \ x\in I_j,\ \ j=1,\ldots,N+1, \tag{2}$$

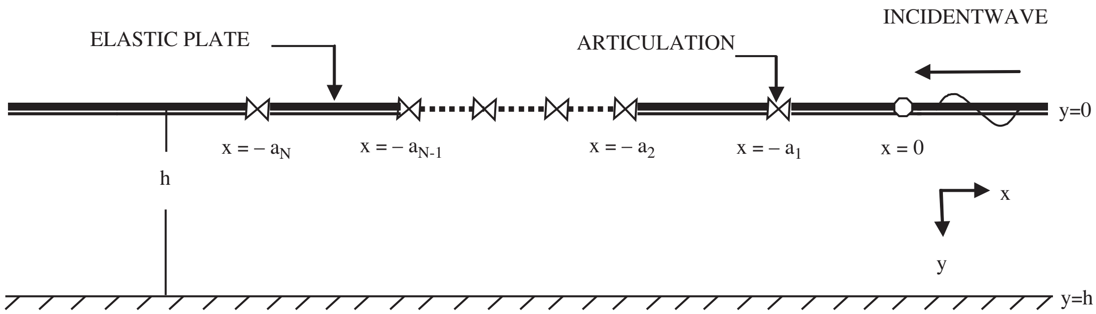

Fig. 1. Schematic diagram for multiple articulated floating elastic plates.

where $D = EI/(\rho_w g - m_s\omega^2)$, $K = \rho_w\omega^2/(\rho_w g - m_s\omega^2)$, $EI = Ed^3/12(1-\nu^2)$ is the rigidity of the plate, E is Young's modulus, $I = d^3/12(1-\nu^2)$, ν is the Poisson ratio, $m_s = \rho_p d$, ρ_p is the density of the elastic plates, ρ_w is the density of water and g is the acceleration due to gravity.

The no-flow condition at the bottom boundary yields

$$\partial_y\phi_j = 0 \quad \text{on } y = h \quad \text{in the case of finite water depth,} \tag{3}$$

$$\phi_j, \nabla\phi_j \to 0 \quad \text{as } y \to \infty \quad \text{in the case of infinite water depth.}$$

In addition, continuity of velocity and pressure across the vertical interface at the articulated edges yield

$$\left.\begin{aligned} \partial_x\phi_j(x+,y) &= \partial_x\phi_{(j+1)}(x-,y) \\ \phi_j(x+,y) &= \phi_{(j+1)}(x-,y) \end{aligned}\right\} \quad \text{at } x = -a_j,\ j = 1,\ldots,N \text{ for all } y. \tag{4}$$

Assuming that the plates are connected by a vertical linear spring and a flexural rotational spring with stiffness k_{33} and k_{55}, respectively, the shear force and the bending moment at the connecting edges $(-a_j, 0), j = 1,\ldots,N$ satisfy the conditions [as in Xia et al. (2000) and Chung and Fox (2005)]

$$EI\partial^3_{yx^2}\phi_j(x+,0) = k_{55}\{\partial^2_{yx}\phi_j(x+,0) - \partial^2_{yx}\phi_{(j+1)}(x-,0)\}, \tag{5a}$$

$$EI\partial^3_{yx^2}\phi_{(j+1)}(x-,0) = k_{55}\{\partial^2_{yx}\phi_j(x+,0) - \partial^2_{yx}\phi_{(j+1)}(x-,0)\}, \tag{5b}$$

$$EI\partial^4_{yx^3}\phi_j(x+,0) = -k_{33}\{\partial_y\phi_j(x+,0) - \partial_y\phi_{(j+1)}(x-,0)\}, \tag{5c}$$

$$EI\partial^4_{yx^3}\phi_{(j+1)}(x-,0) = -k_{33}\{\partial_y\phi_j(x+,0) - \partial_y\phi_{(j+1)}(x-,0)\}. \tag{5d}$$

It may be noted that, if both the stiffness constants k_{33} and k_{55} are absent, then the articulated condition as in Eqs. (5a)–(5d) behaves as free-edge condition. Further, it may be noted that Khabakhpasheva and Korobkin (2002) used similar edge conditions as in (5a) and (5b) in the compound floating structure for attenuating plate vibration. The far field radiation conditions are given by

$$\phi_1(x,y) \sim (\mathrm{e}^{-\mathrm{i}k_0x} + R_N\,\mathrm{e}^{\mathrm{i}k_0x})f_0(y) \quad \text{as } x \to \infty, \quad \phi_{N+1}(x,y) \sim T_N\,\mathrm{e}^{-\mathrm{i}k_0x}f_0(y) \quad \text{as } x \to -\infty, \tag{6}$$

where $f_0(y) = \cosh k_0(h-y)/\cosh k_0h$ in the case of finite water depth, and $f_0(y) = \mathrm{e}^{-k_0y}$ in the case of infinite water depth where k_0 is the wave number of the incident wave satisfying the flexural gravity wave dispersion relation

$$K = (Dk_0^4 + 1)k_0 \tanh k_0h \quad \text{in the case of finite water depth,}$$

$$K = (Dk_0^4 + 1)k_0 \quad \text{in the case of infinite water depth.} \tag{7}$$

The unknown constants R_N and T_N are associated with the amplitude of the reflected and transmitted waves, respectively, in the case of N articulations.

2.2. Wave scattering by single articulated floating elastic plate

In this subsection, we will describe in brief the method of solution for the cases of finite and infinite water depth and shallow water approximation for the flexural gravity wave scattering by a single articulated floating elastic plate.

2.2.1. Finite water depth

The geometry of the present problem is a particular case of the one described in Fig. 1 with $N = 1$. Thus, without any loss of generality, we consider two floating semi-infinite plates which are joined through articulation at $x = 0$. The domain under consideration is divided into two sub-domains $I_j, j = 1,2$, namely I_1 for $(0<x<\infty, 0<y<h)$ and I_2 for $(-\infty<x<0, 0<y<h)$ as in Fig. 2. Keeping in mind the assumptions made in Section 2.1, the spatial velocity potential $\phi_j(x,y), j = 1,2$, satisfies the governing equation (1) along with boundary conditions (2)–(5) as appropriate for the case of finite water depth. Exploiting the geometrical symmetry of the physical problem about the line $x = 0$, $0<y<h$, the boundary value problem defined in the infinite strip $-\infty<x<\infty$, $0<y<h$, is reduced to two boundary value problems in the semi-infinite strip $0<x<\infty$, $0<y<h$, by using the reduced potentials as defined by

$$\varphi(x,y) = \phi_1(x,y) - \phi_2(-x,y) \quad \text{and} \quad \Upsilon(x,y) = \phi_1(x,y) + \phi_2(-x,y), \tag{8}$$

where $\phi_1(x,y)$ and $\phi_2(-x,y)$ represent the velocity potentials in the intervals I_1 and I_2, respectively. The reduced antisymmetric and symmetric potentials $\varphi(x,y)$ and $\Upsilon(x,y)$ satisfy Eqs. (1)–(3) independently. However, the continuity

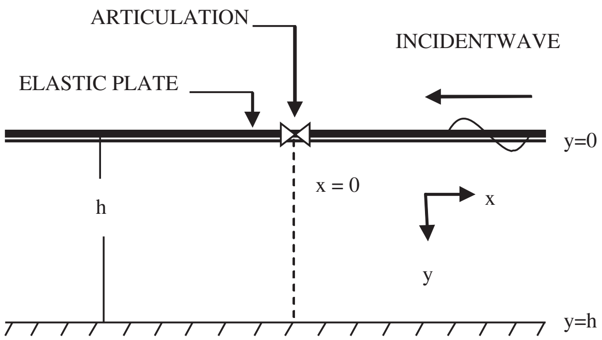

Fig. 2. Schematic diagram for single articulated floating elastic plate.

conditions in Eq. (4) yield

$$\varphi(x,y)=0 \quad \text{and} \quad \partial_x \Upsilon(x,y)=0 \quad \text{at } x=0,\ 0<y<h. \tag{9}$$

Further, the edge conditions (5a)–(5d) in terms of $\varphi(x,y)$ and $\Upsilon(x,y)$ are derived as usual.

Using the expansion formula for the wave-structure interaction problem and the corresponding orthogonal mode-coupling relations as in Sahoo et al. (2001) [see also Manam et al. (2006) and Bhattacharjee et al. (2007)], the reduced potentials $\varphi(x,y)$ and $\Upsilon(x,y)$ are obtained as

$$\varphi(x,y)=\mathrm{e}^{-\mathrm{i}k_0x}f_0(y)+\sum_{n=0}^{II}A_n\,\mathrm{e}^{\mathrm{i}\varepsilon_nk_nx}f_n(y)+\sum_{n=1}^{\infty}A_n\,\mathrm{e}^{-k_nx}f_n(y), \tag{10}$$

$$\Upsilon(x,y)=\mathrm{e}^{-\mathrm{i}k_0x}f_0(y)+\sum_{n=0}^{II}B_n\,\mathrm{e}^{\mathrm{i}\varepsilon_nk_nx}f_n(y)+\sum_{n=1}^{\infty}B_n\,\mathrm{e}^{-k_nx}f_n(y), \tag{11}$$

where $f_n(y)=\cosh k_n(h-y)/\cosh k_nh$ for $n=0,I,II$ and k_n, $n=0,I,II$, satisfies the dispersion relation

$$K=(Dk_n^4+1)k_n\tanh k_nh, \tag{12}$$

with k_0 being a real and positive root, the k_n being purely imaginary and of the form $k_n=\mathrm{i}p_n, p_n>0$ for $n=1,2,\ldots$. Apart from the real positive root and imaginary roots, the dispersion relation in Eq. (12) has four complex roots $k_n, n=I,II,III,IV$, of the form $\pm\alpha\pm\mathrm{i}\beta$ where α, β being real. In the present study, we have considered two complex roots with positive real parts for the sake of boundedness of the solution in the expansion of the reduced velocity potentials as in Eqs. (10) and (11).

The unknown constants A_n and B_n are obtained as

$$\left.\begin{aligned} &A_n=-D\beta_ak_n^3\tanh k_nh/KC_n-\delta_n, \quad \beta_a=\varphi_y(0+,0)=2\mathrm{i}k_0^4EI\tanh k_0h/\Omega_aD,\\ &\Omega_a=\frac{1}{K}\sum_{n=0,I,II,1}^{\infty}\frac{k_n^4}{C_n}(2k_{33}-\mathrm{i}EIk_n^3\varepsilon_n)\tanh^2k_nh, \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} &B_n=\mathrm{i}\varepsilon_nDk_n^2\alpha_s\tanh k_nh/KC_n+\delta_n, \quad \alpha_s=\Upsilon_{yx}(0+,0)=-2EIk_0^3\tanh k_0h/D\Omega_s,\\ &\Omega_s=\frac{\mathrm{i}}{K}\sum_{n=0,I,II,1}^{\infty}\frac{k_n^4}{C_n}(2\mathrm{i}k_{55}+EIk_n\varepsilon_n)\tanh^2k_nh, \end{aligned}\right\} \tag{14}$$

where

$$C_n=\frac{2k_nh(1+Dk_n^4)+(1+5Dk_n^4)\sinh 2k_nh}{4k_n(1+Dk_n^4)\cosh^2k_nh},$$

with

$$\delta_n=\begin{cases}1 & \text{for } n=0,\\ 0 & \text{for } n=I,II,1,2,\ldots\end{cases} \quad \text{and} \quad \varepsilon_n=\begin{cases}1 & \text{for } n=I,0,1,2,\ldots,\\ -1 & \text{for } n=II.\end{cases}$$

Once the constants A_0 and B_0 are determined, the reflection and the transmission coefficients are derived from the relations $K_r = |R_1| = |(B_0 + A_0)/2|$ and $K_t = |T_1| = |(B_0 - A_0)/2|$.

It should be noted that, for specific parameters, often the complex roots are replaced by purely imaginary roots in Eq. (12) and there can be situations where the two roots coalesce and such situations do not correspond to any realistic physical process [see Williams (2006) and Bennetts et al. (2007)]. In the context of the present paper, the author could not come across any such situation and thus the above derivations are based on the assumption that the complex roots of the dispersion relations are unique in nature as described above.

2.2.2. Infinite water depth

In the case of infinite water depth, the geometry of the physical problem is same as in the case of finite water depth with $h \to \infty$. The detailed derivation of the velocity potential in the case of a single articulation in infinite water depth is given in Karmakar and Sahoo (2005). However, the expressions for the potentials are given here for the sake of clarity and completeness, which will be used in the numerical computation for the multiple articulations. Hence, proceeding as in Karmakar and Sahoo (2005), the reduced potentials $\varphi(x,y)$ and $\Upsilon(x,y)$ (defined in a similar manner as in the case of finite water depth) are expanded in terms of appropriate eigenfunctions as given by

$$\varphi(x,y) = \mathrm{e}^{-\mathrm{i}k_0x} f_0(y) + \sum_{n=0}^{II} A_n \,\mathrm{e}^{\mathrm{i}\varepsilon_n k_n x} f_n(y) + \frac{2}{\pi}\int_0^\infty \frac{L(\xi,y)A(\xi)\,\mathrm{e}^{-\xi x}\,d\xi}{\Delta(\xi)}, \tag{15}$$

$$\Upsilon(x,y) = \mathrm{e}^{-\mathrm{i}k_0x} f_0(y) + \sum_{n=0}^{II} B_n \mathrm{e}^{\mathrm{i}\varepsilon_n k_n x} f_n(y) + \frac{2}{\pi}\int_0^\infty \frac{L(\xi,y)B(\xi)\,\mathrm{e}^{-\xi x}\,d\xi}{\Delta(\xi)}, \tag{16}$$

where $L(\xi,y) = \xi(1+D\xi^4)\cos\xi y - K\sin\xi y$, $\Delta(\xi) = \xi^2(1+D\xi^4)^2 + K^2$ and $f_n(y) = \mathrm{e}^{-k_n y}$ for $n = 0, I, II$. In Eqs. (15) and (16), the A_n and B_n are the unknown constants to be determined and $k_n (n = 0, I, II, III, IV)$ are roots of the dispersion relation $K = k_n(Dk_n^4 + 1)$, with k_0 being the real and positive root, k_I and k_{II} are complex conjugate with positive real parts, k_{III} and k_{IV} are complex conjugate with negative real parts. However, in the relation for $\varphi(x,y)$ and $\Upsilon(x,y)$, only two complex roots with positive real parts are taken into account for the sake of boundedness of the solution.

Using the continuity conditions as in Eq. (4) and the edge conditions as in Eqs. (5a)–(5d) along with the orthogonal mode-coupling relations as in Manam et al. (2006), the unknown constants A_n, B_n and the unknown functions $A(\xi)$, $B(\xi)$ are obtained as

$$\begin{aligned} &A_n = -D\beta_a k_n^3/KC_n - \delta_n, \quad A(\xi) = D\xi^3\beta_a, \quad \beta_a = 2\mathrm{i}k_0^4 EI/\Omega_a D, \\ &\Omega_a = \frac{-1}{K}\sum_{n=0}^{II}\frac{k_n^4}{C_n}\{\varepsilon_n EI\mathrm{i}k_n^3 - 2k_{33}\} + \frac{2K}{\pi}\int_0^\infty \frac{\xi^4(EI\xi^3 - 2k_{33})\,d\xi}{\Delta(\xi)}, \end{aligned} \tag{17}$$

$$\begin{aligned} &B_n = \mathrm{i}\varepsilon_n k_n^2 D\alpha_s/KC_n + \delta_n, \quad B(\xi) = -D\xi^2\alpha_s, \quad \alpha_s = -2EIk_0^3/D\Omega_s, \\ &\Omega_s = \frac{\mathrm{i}}{K}\sum_{n=0}^{II}\frac{k_n^4}{C_n}\{\varepsilon_n EIk_n + 2\mathrm{i}k_{55}\} + \frac{2K}{\pi}\int_0^\infty \frac{\xi^4(EI\xi + 2k_{55})\,d\xi}{\Delta(\xi)}, \end{aligned} \tag{18}$$

where $C_n = (1 + 5Dk_n^4)/2K$. Once the coefficients A_0 and B_0 are found, the reflection and transmission coefficients are determined from the relations as given in the case of finite water depth.

2.2.3. Shallow water approximation

Under the assumptions of the linearized shallow water theory, the velocity potentials $\phi_j(x)$ for $j = 1, 2$ and the deflection of the elastic plates $\eta_j(x)$ for $j = 1, 2$ are related as (Sturova, 2006; Ohkusu and Namba, 2004)

$$-\mathrm{i}\omega\eta_j = h\partial_x^2\phi_j. \tag{19}$$

The linearized long wave equation in the plate-covered region for $j = 1, 2$ is obtained as

$$EI\partial_x^6\phi_j + (\rho_w g - m_s\omega^2)\partial_x^2\phi_j + \frac{\rho_w\omega^2}{h}\phi_j = 0. \tag{20}$$

The continuity of energy flux and mass flux across the interface at the articulated edges for $j = 1$ yields (Ohkusu and Namba, 2004)

$$\phi_{(j+1)} = \phi_j \quad \text{and} \quad \partial_x\phi_{(j+1)} = \partial_x\phi_j \quad \text{at } x = 0. \tag{21}$$

Assuming that the plates are articulated through a vertical linear spring and/or a flexural rotational spring with stiffness k_{33} and k_{55}, the edge conditions at $x = 0$ are given by

$$EI\partial_x^4\phi_j(x+) = k_{55}\{\partial_x^3\phi_j(x+) - \partial_x^3\phi_{(j+1)}(x-)\}, \tag{22a}$$

$$EI\partial_x^4\phi_{(j+1)}(x-) = k_{55}\{\partial_x^3\phi_j(x+) - \partial_x^3\phi_{(j+1)}(x-)\}, \tag{22b}$$

$$EI\partial_x^5\phi_j(x+) = -k_{33}\{\partial_x^2\phi_j(x+) - \partial_x^2\phi_{(j+1)}(x-)\}, \tag{22c}$$

$$EI\partial_x^5\phi_{(j+1)}(x-) = -k_{33}\{\partial_x^2\phi_j(x+) - \partial_x^2\phi_{(j+1)}(x-)\}. \tag{22d}$$

The far field radiation condition is the same as in Section 2.1 for $j = 1, 2$. The velocity potential $\phi_j(x), j = 1, 2$ is expanded as given by

$$\phi_j(x) = \begin{cases} (\mathrm{e}^{-\mathrm{i}k_0x} + R_1\,\mathrm{e}^{\mathrm{i}k_0x}) + \sum\limits_{n=I}^{II} R_n^c\,\mathrm{e}^{\mathrm{i}\varepsilon_n k_n x} & \text{for } x > 0, \\ T_1\,\mathrm{e}^{-\mathrm{i}k_0x} + \sum\limits_{n=I}^{II} T_n^c\,\mathrm{e}^{-\mathrm{i}\varepsilon_n k_n x} & \text{for } x < 0, \end{cases} \tag{23}$$

where R_1, T_1, R_n^c, T_n^c, $n = I, II$, are the unknown constants to be determined and k_n, $n = 0, I, II$ satisfies the shallow water flexural gravity wave dispersion relation

$$EIk_n^6 + (\rho_w g - m_s\omega^2)k_n^2 = \frac{\rho_w\omega^2}{h}, \quad n = 0, I, II. \tag{24}$$

It may be noted that the dispersion relation as in Eq. (24) has two real roots $\pm k_0$ with $k_0 > 0$ and four complex roots of the form $k_n = \pm\alpha \pm \mathrm{i}\beta$ for $n = I, II, III, IV$. In the present study, terms involving the negative real root and the two complex roots having negative real parts are neglected due to the boundedness of the solution. The six unknown constants R_1, T_1, R_n^c, T_n^c with $n = I, II$ are associated with the amplitude of the waves and are determined by using the conditions as in Eqs. (21) and (22a)–(22d) which yield a linear system of six algebraic equations. Once the unknowns R_1 and T_1 are obtained, the reflection and transmission coefficients are evaluated from the relations $K_r = |R_1|$ and $K_t = |T_1|$.

2.3. *Wave scattering by multiple articulations*

In the present subsection, we shall extend the results of a single articulation to N articulations by applying the wide-spacing approximation method and direct method. The general procedure for the wide-spacing approximation for multiple articulations is the same for all water depths and depends on the solution procedure for a single articulation. However, in the direct method, solution procedures are very complex in nature in both the cases of finite and infinite water depths. Hence, the results by the direct method are discussed only in the case of shallow water approximation.

2.3.1. *Wide-spacing approximation for N articulations*

The general boundary value problem is same as defined in Section 2.1. Here, the flexural gravity wave experiences partial reflection and transmission at the articulated joints, located at $x = -a_j$ for $j = 1, 2, \ldots, N$. It is assumed that the distance between two consecutive articulations is much larger than the wavelength of the incident plane progressive wave, i.e., $|a_{j+1} - a_j| \gg \lambda$ for $j = 1, 2, \ldots, N-1$, where λ is the incident wavelength [as in Dingemans (1997) and McIver (1986)] to ensure that the evanescent modes do not contribute to the solution. Thus, the local effects produced during the interaction of the incident wave with one of the articulated joints do not affect the subsequent interactions. Assuming that the articulated edges are placed widely apart, the asymptotic form of the velocity potential ϕ_j for $j = 1, 2, \ldots, N+1$ far away from the articulations in the respective regions are given by

$$\begin{aligned} &\phi_1 \sim \mathrm{e}^{-\mathrm{i}k_0x}f_0(y) + R_N\,\mathrm{e}^{\mathrm{i}k_0x}f_0(y), \quad -a_1 < x < \infty, \\ &\phi_{j+1} \sim A_j\,\mathrm{e}^{-\mathrm{i}k_0x}f_0(y) + B_j\,\mathrm{e}^{\mathrm{i}k_0x}f_0(y), \quad -a_{j+1} < x < -a_j, \; j = 1, 2, \ldots, N-1, \\ &\phi_{N+1} \sim T_N\,\mathrm{e}^{-\mathrm{i}k_0x}f_0(y), \quad -\infty < x < -a_N. \end{aligned} \tag{25}$$

Equating the left and right going components of the propagating waves at the articulated edges $x = -a_j$ for $j = 1, 2, \ldots, N$ with the amplitudes of the reflected and transmitted waves in the prescribed region, a system of $2N$ linear equations associated with $2N$ unknowns $R_N, T_N, A_j, B_j, j = 1, 2, \ldots, N-1$ are obtained as given by

$$\begin{aligned} &R_N\,\mathrm{e}^{-\mathrm{i}k_0a_1} = R_1\,\mathrm{e}^{\mathrm{i}k_0a_1} + B_1T_1\,\mathrm{e}^{-\mathrm{i}k_0a_1}, \quad A_j\,\mathrm{e}^{-\mathrm{i}k_0a_j} = T_1A_{j-1}\,\mathrm{e}^{\mathrm{i}k_0a_j} + B_jR_1\,\mathrm{e}^{-\mathrm{i}k_0a_j},\\ &B_j\,\mathrm{e}^{-\mathrm{i}k_0a_{j+1}} = A_jR_1\,\mathrm{e}^{\mathrm{i}k_0a_{j+1}} + T_1B_{j+1}\,\mathrm{e}^{-\mathrm{i}k_0a_{j+1}}, \quad T_N\,\mathrm{e}^{\mathrm{i}k_0a_N} = A_{N-1}T_1\,\mathrm{e}^{\mathrm{i}k_0a_N}, \end{aligned} \tag{26}$$

with $A_0 = 1$ and $B_N = 0$. Here, R_1 and T_1 corresponds to the amplitude of the reflected and transmitted waves for single articulation. Solving the above system of equations, the reflection and transmission coefficients $K_r = |R_N|$ and $K_t = |T_N|$ for N articulations are obtained.

2.3.2. Direct method for N articulations in the case of finite water depth

The velocity potentials $\phi_j(x, y)$ in each of the $(N+1)$ regions in the case of finite water depth are expressed in terms of appropriate eigenfunctions as given by

$$\phi_j = \begin{cases} \mathrm{e}^{-\mathrm{i}k_0(x+a_1)}f_0(y) + R_N\,\mathrm{e}^{\mathrm{i}k_0(x+a_1)}f_0(y) + \sum\limits_{n=I}^{II} R_n^c\,\mathrm{e}^{\mathrm{i}\varepsilon_nk_n(x+a_1)}f_n(y) + \sum\limits_{n=1}^{\infty} R_n^e\,\mathrm{e}^{-k_n(x+a_1)}f_n(y) \quad \text{for } x > -a_1,\\ \sum\limits_{n=0,I}^{II}(A_n^j\cos k_nx + B_n^j\sin k_nx)f_n(y) + \sum\limits_{n=1}^{\infty}\left\{A_n^j\dfrac{\cosh k_nx}{\cosh k_n\delta a_j} + B_n^j\dfrac{\sinh k_nx}{\sinh k_n\delta a_j}\right\}f_n(y) \quad \text{for } x\in I_j, j = 2,3,\ldots,N,\\ T_N\,\mathrm{e}^{-\mathrm{i}k_0(x+a_N)}f_0(y) + \sum\limits_{n=I}^{II} T_n^c\,\mathrm{e}^{-\mathrm{i}\varepsilon_nk_n(x+a_N)}f_n(y) + \sum\limits_{n=1}^{\infty} T_n^e\,\mathrm{e}^{k_n(x+a_N)}f_n(y) \quad \text{for } x < -a_N, \end{cases} \tag{27}$$

where $I_j = (-a_j, -a_{j-1})$ for $j = 2, 3, \ldots, N$ with $\delta a_j = dist(I_j)$, $\varepsilon_n = 1$ for $n = I$, $\varepsilon_n = -1$ for $n = II$ and R_N, T_N, R_n^c, T_n^c for $n = I, II$, R_n^e, T_n^e for $n = 1, 2, \ldots$, A_n^j, B_n^j for $n = 0, I, II, 1, \ldots$, are the unknown constants to be determined. The eigenfunctions $f_n(y)$ and the associated eigenvalues k_n are the same as defined in Section 2.2.1 in the case of finite water depth. The infinite sums are truncated up to a finite number m(say) in the expansion of the velocity potentials in each regions as defined in Eq. (27). Using the continuity of pressure and velocity as in Eq. (4), the velocity potentials ϕ_j as in Eq. (27) along with finite number of terms m(say), the edge conditions as in Eqs. (5a)–(5d) and the orthogonal mode-coupling relation as in Manam et al. (2006), a system of $2N(m+4)$ linear equations can be obtained for the determination of the unknown constants associated with the velocity potential and the edge conditions. The unknowns associated with the edge conditions will be similar to the ones defined in Evans and Porter (2003) and in Bhattacharjee et al. (2007). Once the unknowns R_N and T_N are obtained, the reflection and transmission coefficients are derived from the relations $K_r = |R_N|$ and $K_t = |T_N|$. It may be further noted that K_r and K_t satisfy the energy relation $K_r^2 + K_t^2 = 1$, which will be used to check the numerical results. However, the details are deferred here in the context of the present paper.

2.3.3. Direct method for N articulations in the case of infinite water depth

The velocity potentials $\phi_j(x, y)$ in each of the $(N+1)$ regions in the case of infinite water depth are expressed in terms of appropriate eigenfunctions as given by

$$\phi_j = \begin{cases} \mathrm{e}^{-\mathrm{i}k_0(x+a_1)}f_0(y) + R_N\,\mathrm{e}^{\mathrm{i}k_0(x+a_1)}f_0(y) + \sum\limits_{n=I}^{II} R_n^c\,\mathrm{e}^{\mathrm{i}\varepsilon_nk_n(x+a_1)}f_n(y)\\ \quad +\dfrac{2}{\pi}\displaystyle\int_0^{\infty}\dfrac{L(\xi,y)R(\xi)\,\mathrm{e}^{-\xi x}\,d\xi}{\Delta(\xi)} \quad \text{for } x > -a_1,\\ \sum\limits_{n=0,I}^{II}(A_n^j\cos k_nx + B_n^j\sin k_nx)f_n(y) + \dfrac{2}{\pi}\displaystyle\int_0^{\infty}\dfrac{L(\xi,y)(A^j(\xi)\mathrm{e}^{-\xi x} + B^j(\xi)\mathrm{e}^{\xi x})\,d\xi}{\Delta(\xi)}\\ \quad \text{for } x\in I_j, j = 2,3,\ldots,N,\\ T_N\,\mathrm{e}^{-\mathrm{i}k_0(x+a_N)}f_0(y) + \sum\limits_{n=I}^{II} T_n^c\,\mathrm{e}^{-\mathrm{i}\varepsilon_nk_n(x+a_N)}f_n(y) + \dfrac{2}{\pi}\displaystyle\int_0^{\infty}\dfrac{L(\xi,y)T(\xi)\,\mathrm{e}^{\xi x}\,d\xi}{\Delta(\xi)}\\ \quad \text{for } x < -a_N, \end{cases} \tag{28}$$

where $L(\xi, y)$, $\Delta(\xi)$, k_n and $f_n(y)$ for $n = 0, I, II$, $j = 1, 2, \ldots, N+1$ are same as defined in Section 2.2.2 in the case of infinite water depth with I_j, δa_j and ε_n are same as defined in the Section 2.3.2. The constants R_N, R_n^c, T_N, T_n^c for $n = I, II$, A_n^j, B_n^j for $n = 0, I, II$, and the functions $R(\xi)$, $T(\xi)$, $A^j(\xi)$ and $B^j(\xi)$ are to be determined. Using the velocity potentials in each region as defined in Eq. (28) and utilizing the continuity of pressure and velocity as in Eq. (4), the edge conditions as in Eqs. (5a)–(5d) and the orthogonal mode-coupling relation as in Manam et al. (2006), the unknown constants and the unknown functions are to be determined. It may be noted that, in this case, this will lead to a system of integral equations for the determination of the $8N$ unknown constants and $2N$ unknown functions associated with the velocity potentials as defined in Eq. (28) and the prescribed edge conditions. The details require special attention as a separate problem and are deferred here in the context of the present paper.

2.3.4. Direct method for N articulations based on shallow water approximation

The physical problem is similar to the one described in Section 2.1 in the case of wave scattering by multiple articulated floating elastic plates in water of finite depth. The velocity potential $\phi_j(x), j = 1, 2, \ldots, N+1$ are expanded as given by

$$\phi_j(x) = \begin{cases} (\mathrm{e}^{-\mathrm{i}k_0 x} + R_N\, \mathrm{e}^{\mathrm{i}k_0 x}) + \sum_{n=I}^{II} R_n^c\, \mathrm{e}^{\mathrm{i}\varepsilon_n k_n x} & \text{for } x > -a_1, \\ A_0^j\, \mathrm{e}^{-\mathrm{i}k_0 x} + B_0^j\, \mathrm{e}^{\mathrm{i}k_0 x} + \sum_{n=I}^{IV} A_n^j\, \mathrm{e}^{-\mathrm{i}k_n x} & \text{for } x \in I_j, j = 2, 3, \ldots, N, \\ T_N\, \mathrm{e}^{-\mathrm{i}k_0 x} + \sum_{n=I}^{II} T_n^c\, \mathrm{e}^{-\mathrm{i}\varepsilon_n k_n x} & \text{for } x < -a_N, \end{cases} \tag{29}$$

where $I_j = (-a_j, -a_{j-1})$ for $j = 2, \ldots, N$ with R_N, T_N, R_n^c, T_n^c, $n = I, II$ and $A_0^j, B_0^j, A_n^j, n = I, II, III, IV$, are the unknown constants to be determined, and k_n satisfies the shallow water flexural gravity wave dispersion relation as in Eq. (24) having four complex roots $k_n, n = I, II, III, IV$ of the form $\pm\alpha \pm \mathrm{i}\beta$ and two real roots $\pm k_0$, which represent the progressive wave modes. Using the continuity conditions (21) and the articulated edge conditions (22a)–(22d) at $x = -a_j, j = 1, 2, \ldots, N$, we have a system of $6N$ linear algebraic equations to solve for $6N$ unknowns in the case of N articulations. Once the unknowns R_N and T_N are obtained, the reflection and transmission coefficients are evaluated from the relations $K_r = |R_N|$ and $K_t = |T_N|$.

3. Numerical results and discussion

The numerical results for the reflection coefficient K_r are obtained using the wide-spacing approximation method for both finite and infinite water depths to analyse the scattering of flexural gravity waves by floating elastic plates having multiple articulations. On the other hand, in the case of shallow water approximation, both the direct method and wide-spacing approximation method are used for analysing the numerical results. The results for a single, double and four articulations are presented here for different values of the nondimensional plate length $\delta a_j/d$ with $\delta a_j = |a_{j+1} - a_j|$ for $j = 1, 2, \ldots, N-1$, water depth h/d, nondimensional Young's modulus $E^1 = E/\rho_w g d$, nondimensional vertical linear spring stiffness $k_{33}^1 = k_{33}/\rho_w g d^2$, nondimensional flexural rotational spring stiffness $k_{55}^1 = k_{55}/\rho_w g d^4$ and nondimensional wave period $\tau^1 = \tau\sqrt{g/d}$, in each of the three cases of finite water depth, infinite water depth and shallow water approximation. The numerical values of the parameters that are kept fixed during the computations are $d = 1.0\,\mathrm{m}$, $\rho_w = 1025\,\mathrm{kg\,m^{-3}}$, $\rho_p/\rho_w = 0.9$, $v = 0.3$, $g = 9.8\,\mathrm{m\,s^{-2}}$, $E^1 = 4.9776 \times 10^5$ with $h/d = 100.0$ for finite water depth and $h/d = 10.0$ for shallow water approximation. The computational results for all the cases are checked with the energy relation $K_r^2 + K_t^2 = 1$. Hence, to avoid repetition, in most of the cases only the results for the reflection coefficients K_r are analysed.

Fig. 3 shows the variation of reflection coefficient K_r versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in the case of single articulation. In this case it is observed that the reflection coefficient attains a minimum and then increases to a certain value after decreasing to zero with the increase in the nondimensional wave period τ^1. The results show same pattern for all the cases of finite water depth, infinite water depth and shallow water approximation. However, for higher values of τ^1, the reflection coefficient for finite and infinite water depth approaches to zero faster than the shallow water approximation.

Fig. 4 shows the variation of reflection coefficient K_r versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $k_{33}^1 = 10.0$, $k_{55}^1 = 10.0$ and $\delta a_1/d = 100.0$ in the case of two articulations. In this case, it is observed that the wave reflection is less in-between the nondimensional wave period $10 < \tau^1 < 17$. Further, the number of zeros is greater in-between the nondimensional wave period $0 < \tau^1 < 17$ for infinite water depth. Comparison shows that for $0 < \tau^1 < 15$, the reflection coefficient K_r is higher for finite water depth, whereas for $15 < \tau^1 < 40$ it is higher for infinite water depth. However, the reflection coefficient maintains intermediate values in the case of shallow water approximation. It may also be noted that the number of zeros in the reflection coefficient for the two articulations is greater than the zeros observed in the case of a single articulation. This shows that the resonating pattern increases with the increase in articulations. This resonating pattern in the reflection coefficients can be referred to as Bragg resonance which generally occurs in water wave problems involving periodic structures, as described in Bennetts et al. (2009) and Marchenko and Voliak (1997).

In Fig. 5, the reflection coefficient K_r is plotted versus nondimensional wavenumber k_0d for various values of k_{33}^1 and k_{55}^1 with $h/d = 40.0$ and $\delta a_1/d = 40.0$ in the case of two articulations for water of finite depth. It is observed that, for

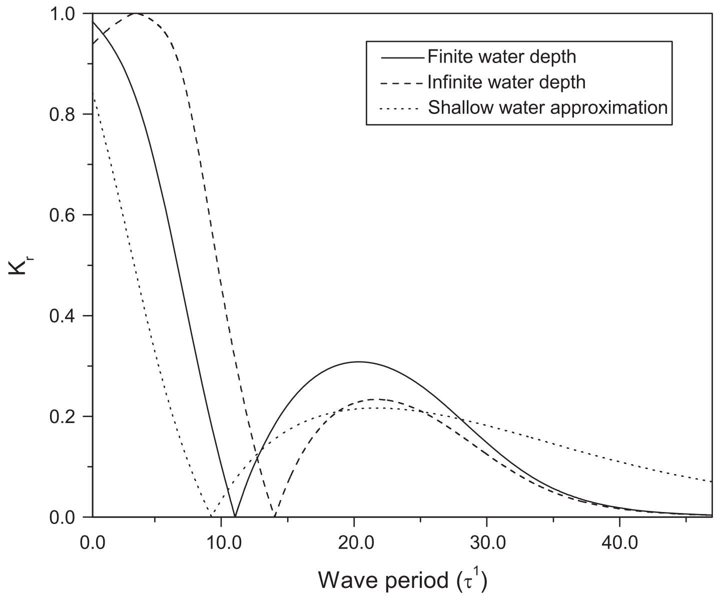

Fig. 3. Reflection coefficient K_r versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in the case of single articulation.

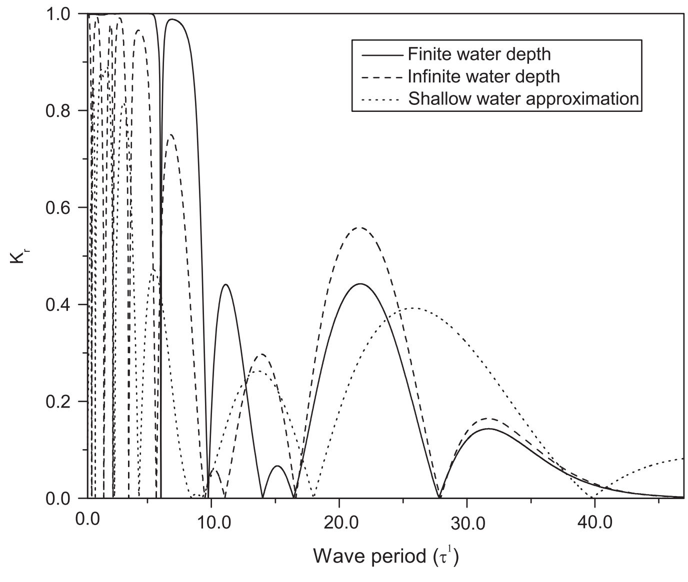

Fig. 4. Reflection coefficient K_r versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $\delta a_1/d = 100.0$, $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in the case of two articulations.

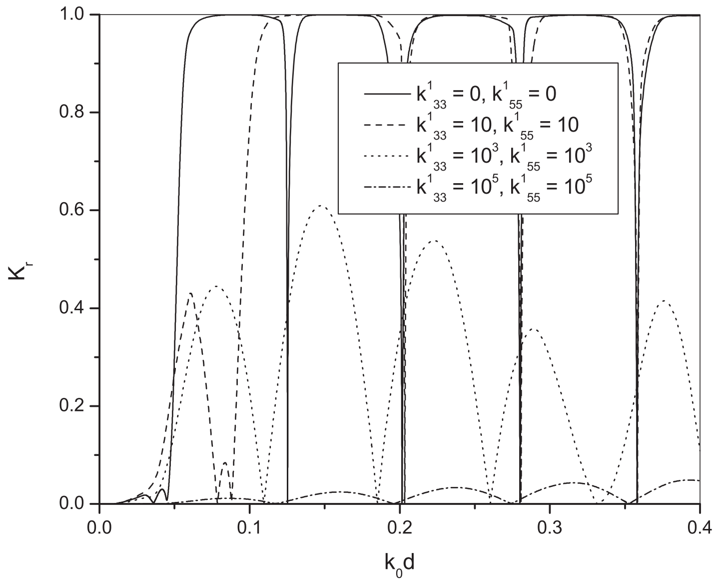

Fig. 5. Reflection coefficient K_r versus nondimensional wave number k_0d for various values of k^1_{33} and k^1_{55} with $\delta a_1/d = 40.0$ and $h/d = 40.0$ in the case of two articulations for finite water depth.

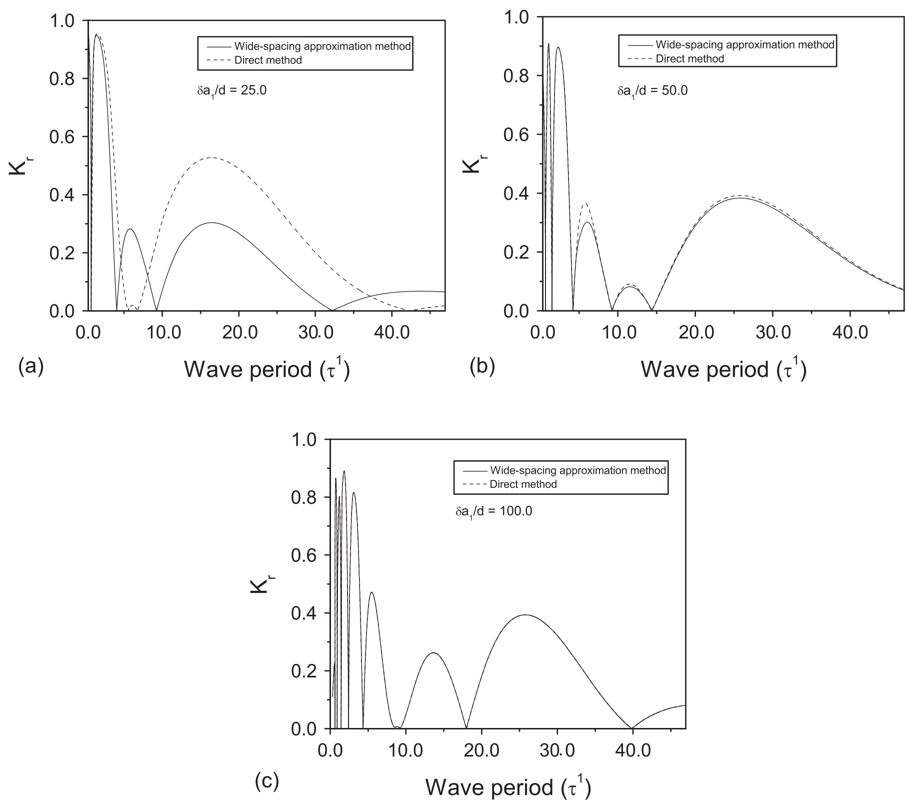

Fig. 6. Reflection coefficient K_r versus nondimensional wave period τ^1 using direct method and wide-spacing approximation method with $h/d = 10.0$, $k^1_{33} = 10.0$ and $k^1_{55} = 10.0$ in the case of two articulations for shallow water approximation.

$k_{33}^1 = 0$ and $k_{55}^1 = 0$, the result exactly matches with the result obtained by Evans and Porter (2004). Further, as k_{33}^1 and k_{55}^1 increases, K_r decreases and for sufficiently higher values of k_{33}^1 and k_{55}^1, K_r becomes zero. This suggests that as the connector stiffness increases, the plates behave as a fully welded and continuous single plate. The resonating pattern in the reflection coefficient for lower values of the stiffness constants k_{33}^1 and k_{55}^1 is prominent and strong as compared to higher values of the stiffness constants.

The comparison using the direct method and the wide-spacing approximation method for the reflection coefficient K_r is plotted versus nondimensional wave period τ^1 for various values of $\delta a_1/d$ with $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in Fig. 6 in the case of shallow water approximation. It is evident from the figure that the results using the direct method and wide-spacing approximation method coincide when $\delta a_1/d \geq 100.0$. This suggests that, in order to study the wave interaction with multiple articulated floating elastic plates, we need to keep the articulated plate of length $\delta a_1/d \geq 100.0$ so as to get satisfactory results. In this case it is found that the number of zeros in the reflection coefficient is larger in-between the nondimensional wave periods $0 < \tau^1 < 17$. This shows that for waves with short wavelength the effect of wave reflection on the articulated joint is more dominant. It is also observed that for $\delta a_1/d = 25.0$, the direct method and the wide-spacing approximation method coincide for $\tau^1 < 2.5$ (which corresponds to $\lambda/d < 25$); for $\delta a_1/d = 50.0$, both the direct method and wide-spacing approximation method coincide for $\tau^1 < 5.0$ (which corresponds to $\lambda/d < 50$). However, for $\delta a_1/d = 100.0$, both methods coincide for all wave periods τ^1. These observations verify that the assumption for the wide-spacing approximation does not valid for smaller plate length, whereas for larger plate length the result using both these methods coincide.

The reflection coefficient K_r is plotted versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $k_{33}^1 = 10.0$, $k_{55}^1 = 10.0$ in Fig. 7 for $\delta a_1/d = 100.0$, $\delta a_2/d = 200.0$ and $\delta a_3/d = 100.0$ in the case of four articulations. It is observed that the number of zeros in K_r increases significantly as the number of articulations increases, compared to the case of double articulations. However, as observed earlier, K_r is smaller for sufficiently higher values of nondimensional wave periods. The reflection coefficient is higher for finite water depth for nondimensional wave periods $0 < \tau^1 < 15$, but for $15 < \tau^1 < 45$ the reflection coefficient is higher in the case of infinite water depth, which is similar to the case of two articulations. Further, the number of zeros in the reflection coefficient is maximum in the case of water of infinite depth and minimum in the case of shallow water approximation.

In Fig. 8, the reflection coefficient K_r is plotted versus nondimensional wave period τ^1 for various values of $\delta a_2/d$ with $h/d = 100.0$, $\delta a_j/d = 100.0$, $j = 1, 3$, $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in the case of four articulations for finite water depth. In this case it is observed that, as the plate length $\delta a_2/d$ increases, there is an increase in the resonating pattern of the reflection coefficient for $0 < \tau^1 < 15$. This shows that, for waves of short period, the resonating pattern increases with the increase in the distance between the articulated plates.

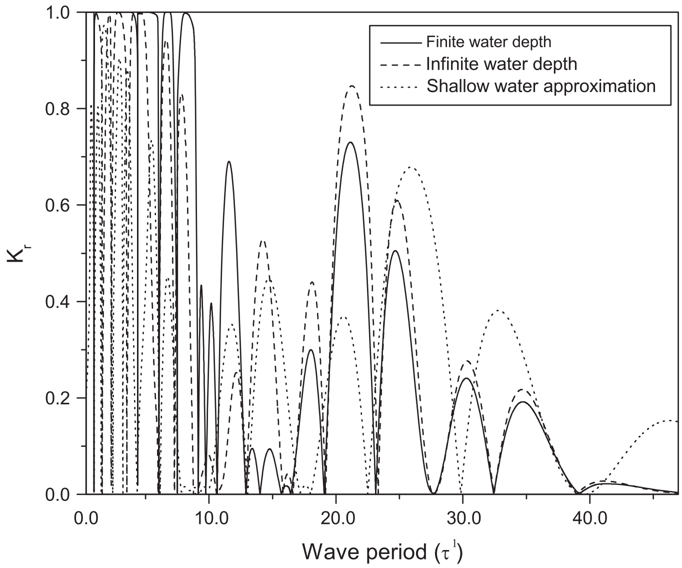

Fig. 7. Reflection coefficient K_r versus nondimensional wave period τ^1 for finite water depth, infinite water depth and shallow water approximation with $\delta a_j/d = 100.0$, $j = 1, 3$, $\delta a_2/d = 200.0$, $k_{33}^1 = 10.0$ and $k_{55}^1 = 10.0$ in the case of four articulations.

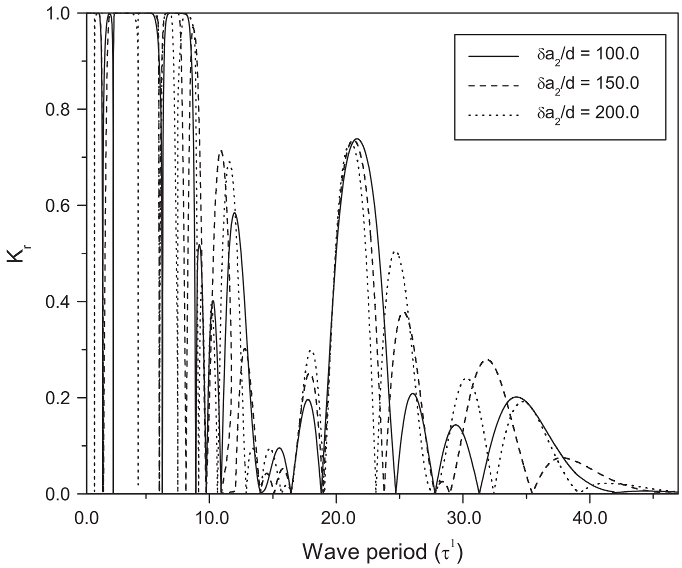

Fig. 8. Reflection coefficient K_r versus nondimensional wave period τ^1 for various values of $\delta a_2/d$ with $h/d = 100.0$, $\delta a_j/d = 100.0$, $j = 1, 3$, $k^1_{33} = 10.0$ and $k^1_{55} = 10.0$ in the case of four articulations for finite water depth.

4. Conclusion

In the present paper, flexural gravity wave scattering by multiple articulated floating elastic plates in the case of water of finite depth, infinite depth and shallow water approximation is compared by analysing the reflection coefficient. It is observed that the number of zeros in the reflection coefficient is maximum in the case of infinite water depth and minimum in case of the shallow water approximation. The resonating pattern in the reflection coefficient increases with the increase in the number of articulations, which is referred to as Bragg resonance in the case of periodic articulated floating elastic plates. It is also found that multiple plates having free edges show high resonating pattern in the reflection coefficient compared to articulated plates with high stiffness constant. The analysis shows that if both the vertical linear springs and the flexural rotation springs are operating simultaneously, there exist limiting values for both the stiffness constants, beyond which the multiple articulated plates behave like a single continuous plate.

A comparison between the direct method and the wide-spacing approximation method in the case of shallow water approximation shows that the results for the reflection coefficients coincides for the nondimensional plate length greater than or equal to 100 for all wave periods, as discussed in Section 3. In other words, for large plates of length larger than 100 times that of plate thickness, the results by both methods agree well irrespective of the wave period.

The work done can be extended to study the oblique flexural gravity wave scattering due to multiple articulations and due to multiple abrupt variations in bottom topography. The present work is expected to be of interest to Naval architects and ocean engineers involved in the design of large floating structures and polar scientists involved in the analysis of wave-ice interaction problems.

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